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ON ORGANISING SALES TOUR UNDER COMPETITIVE CONDITIONS FOR OPTIMAL PROFITS

(*A Mathematical Model*)

UNDER competitive conditions planning to reduce the distribution cost is no less an important effort for maximisation of profits. The usual way of reducing the cost of distribution is to determine the optimal permutation of the various business places which a salesman is likely to visit such that the cost involved is reduced to minimum. The problem is faced in most of the organizations who are concerned with the deliveries of goods by rail/road transport, with the arranging of tours of persons who are expected to visit several places and return to the starting station, with the utilization of transport available for the best advantage of the organization. The solution to such problems is essentially difficult in nature and thus has attracted many researchers. A hypothetical situation of general interest which has not drawn attention,. could be put as follows :

"A salesman is required to visit certain number of places (n) for making his business after starting from place A_0. The associated cost of travelling (C) from place A_i to place A_j is C_{ij} (where $i=0, 1, 2, \ldots, n$; $j=1, 2, \ldots, n$; $i \neq j$). The expected profit at place j is P_j ($j=1, 2, \ldots, n$). It is also known to the salesman that due to high competition in the market, the expected profit P_j at place j is reduced and the reduction in profit to be earned at place j has a relation with delay in making a visit to that place (as delay causes satisfaction of wants in part at least by other competitors who have given priority to that place in their tour programme). Let the reduction factor be p_k at place P_j if place j is visited after having visited k-1 places where ken, $p_1=1$ and $p_1>p_2>\ldots p_n> 0$."

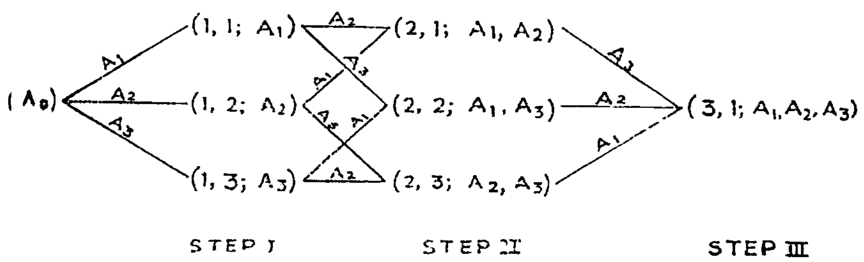
In this situation, the problem a businessman would face is that of determining the tour which results in a maximum profit to him, if he is not allowed to visit a place second time unless he has visited all the "n" places.

In this paper firstly all possible schedules have been studied and later a mathematical formulation of the problem is given. A numerical example is also given at the end to illustrate various steps involved in the computation.

The Possible Schedules of the Tour Problem

A flow chart including all possible schedules for a 'three-place' problem is given below where the starting place and the other three places to be visited are denoted by A_0 and A_i $(i=1,2,3)$ respectively.

FIGURE I



The symbols used in the above mentioned figure have the following meanings:

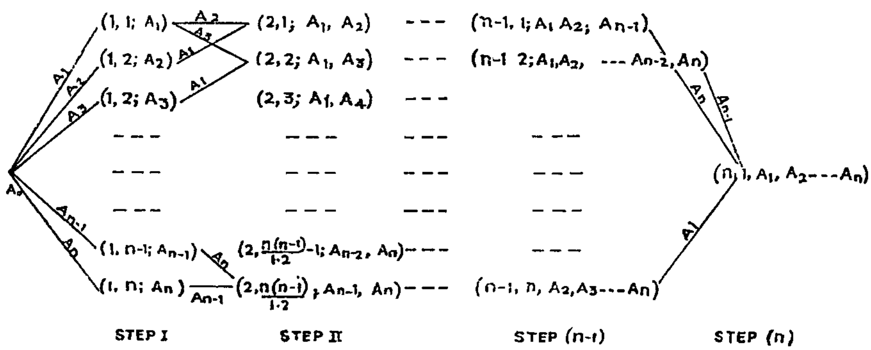
(*i*) A_i on a line indicates that one is heading towards place A_i.

(*ii*) 'Steps' indicate the number of places visited, say for example, all lines entering the Step II indicate that the salesman has visited only place one and now approaching to visit place two.

(*iii*) Figures in parenthesis, for example on Step II $(2,2; A_1, A_3)$, indicate that at second step the second possibility could be that of selecting A_1 first and the A_3 or vice versa.

This presentation of all possible schedules for three places given in Fig. I can be generalised to n places as given in Fig. II.

FIGURE II



From Fig. II we can derive the following information which can be verified from Fig. I. for n=3.

(*i*) In an n-dimensional problem the vertices are 2^n including the starting place.

(*ii*) Each vertex is the meeting point of exactly n lines.

(*iii*) The total number of lines in an n-dimensional problem would be $n.\ 2^{n-1}$.

(*iv*) The number of vertices at step r is given by $\binom{n}{r}$.

(*v*) The number of incoming lines at each vertex in step r are exactly r and out going lines are (n−r).

(*vi*) The number of possible paths in n-dimensional problem is $\lfloor n$ and these paths correspond to the n different permutations of n elements.

This information will be of use in developing a mathematical model.

Mathematical Presentation of the Problem

Dofine,

$V_{k,h}$ = the h^{th} vertix at step k where $k \in n$.

$F(A_i, V_{k,h})$ = Max. expected profit by employing an opitmal policy of visiting k places when the last place visited is A_i.

Using these definitions, we can obtain the following functional relation.

$$F(A_j, V_{k+1,m}) = \max_{i \in V_{k,h}} \left[p_{k+1} P_j - (C_{ij}) + F(A_i, V_{k,h}) \right] \quad \text{...(I)}$$

The solution of recurrence relation I can be iniliated by II below:

$F(A_0, V_{ih}) = (P_i - C_{0i})$

The solution of this equation is illustrated by giving a numerical example.

Numerical Illustration

We determine a tour for the following values of C_{ij}, P_j and p_k given below in Table I through III.

TABLE I

Table giving value of C_{ij} where $C_{ij} = C_{ji}$

j	1	2	3	4
0	12	6	4	1
1	—	5	7	9
2	—	—	8	15
3	—	—	—	10

TABLE II

Table giving profit at various places

Place A_i	1	2	3	4
Profit P_j	120	140	200	150

TABLE III

Table giving values of reduction factor

k	1	2	3	4
p_k	1	·9	·8	·6

Using equations I and II, we get the following values of $F(A_o, V_{1,h})$ and $F(A_i, V_{k,h})$ for k=2, 3 and 4. These are tabulated below in Tables IV through VII.

TABLE IV

$F(A_o, V_{i,h})$	*Path*
108	A_0A_1
134	A_0A_2
196	A_0A_3
149	A_0A_4

TABLE V

$F(A_i, V_{2,h})$	*Path*
229	$A_0A_1A_2$
237	$A_0A_2A_1$
281	$A_0A_1A_3$
297	$A_0A_3A_1$
234	$A_0A_1A_4$
248	$A_0A_4A_1$
306	$A_0A_2A_3$
314	$A_0A_3A_2$
254	$A_0A_2A_4$
260	$A_0A_4A_2$
321	$A_0A_3A_4$
319	$A_0A_4A_3$

TABLE VI

$F(A_i, V_{3,h})$	*Path*
390	$A_0A_2A_1A_3$
418	$A_0A_3A_1A_2$
305	$A_0A_3A_2A_1$
348	$A_0A_2A_1A_4$
355	$A_0A_4A_1A_2$
351	$A_0A_4A_3A_1$
408	$A_0A_3A_1A_4$
401	$A_0A_4A_1A_2$
408	$A_0A_4A_3A_4$
419	$A_0A_3A_2A_4$
412	$A_0A_4A_2A_3$
423	$A_0A_4A_3A_2$

TABLE VII

$F(A_i, V_{4,1})$	*Path*
493	$A_0A_3A_1A_2A_4$
467	$A_0A_4A_1A_2A_3$
487	$A_0A_4A_3A_1A_2$
490	$A_0A_4A_3A_2A_1$

The profit, the businessman would earn in the round trip for different combinations is given below :

$$F(A_i, V_{i,1}) - C_{i,0}$$
$$493-1=492$$
$$467-4=463$$
$$487-6=481$$
$$490-12=478$$

Therefore, the optimal path would be to select the one which corresponds to maximum profit i.e. 492 units of money. This path is given by,

$$A_0 \to A_3 \to A_1 \to A_2 \to A_4 \to A_0.$$

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